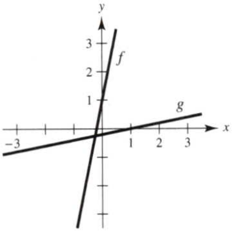
Section 5.3 (page 349)

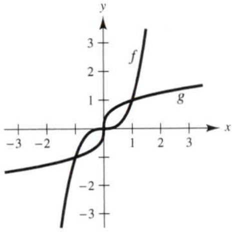
1. (a) $f(g(x)) = 5[(x - 1)/5] + 1 = x$
$g(f(x)) = [(5x + 1) - 1]/5 = x$

(b)



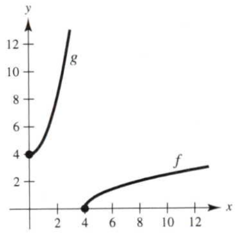
3. (a) $f(g(x)) = \left(\sqrt[3]{x}\right)^3 = x;\ g(f(x)) = \sqrt[3]{x^3} = x$

(b)



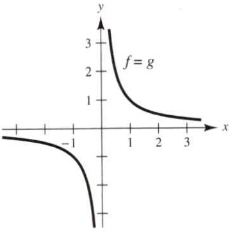
5. (a) $f(g(x)) = \sqrt{x^2 + 4 - 4} = x;$
$g(f(x)) = \left(\sqrt{x - 4}\right)^2 + 4 = x$

(b)



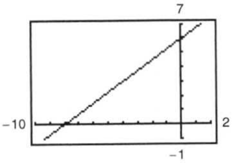
7. (a) $f(g(x)) = \dfrac{1}{1/x} = x;\ g(f(x)) = \dfrac{1}{1/x} = x$

(b)



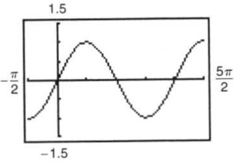
9. c **10.** b **11.** a **12.** d

13.



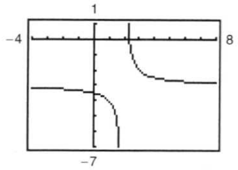
One-to-one, inverse exists.

15.



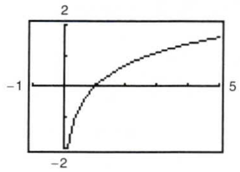
Not one-to-one, inverse does not exist.

17.



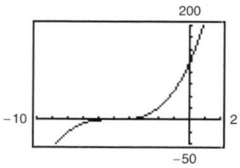
One-to-one, inverse exists.

19.



One-to-one, inverse exists.

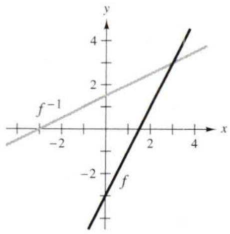
21.



One-to-one, inverse exists.

23. (a) $f^{-1}(x) = (x + 3)/2$

(b)

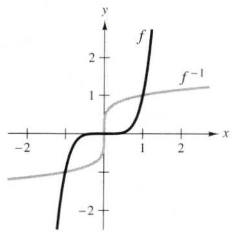


(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: all real numbers
Range of f and f^{-1}: all real numbers

25. (a) $f^{-1}(x) = x^{1/5}$

(b)

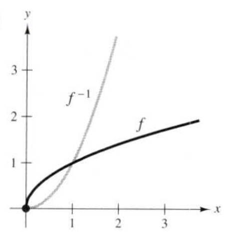


(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: all real numbers
Range of f and f^{-1}: all real numbers

27. (a) $f^{-1}(x) = x^2,\ x \geq 0$

(b)

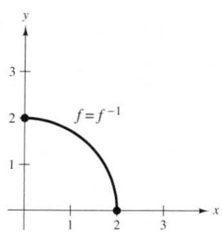


(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: $x \geq 0$
Range of f and f^{-1}: $y \geq 0$

29. (a) $f^{-1}(x) = \sqrt{4 - x^2}$, $0 \leq x \leq 2$

(b)

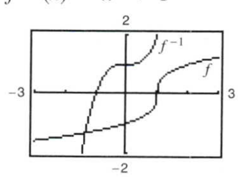


(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: $0 \leq x \leq 2$
Range of f and f^{-1}: $0 \leq y \leq 2$

31. (a) $f^{-1}(x) = x^3 + 1$

(b)

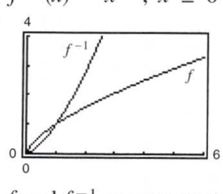


(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: all real numbers
Range of f and f^{-1}: all real numbers

33. (a) $f^{-1}(x) = x^{3/2},\ x \geq 0$

(b)

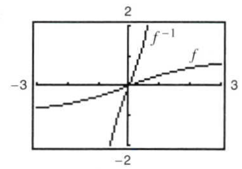


(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: $x \geq 0$
Range of f and f^{-1}: $y \geq 0$

35. (a) $f^{-1}(x) = \sqrt{7}x/\sqrt{1 - x^2}$, $-1 < x < 1$

(b)



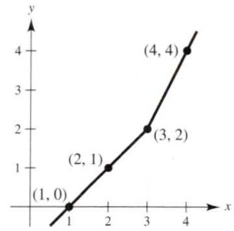
(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f: all real numbers
Domain of f^{-1}: $-1 < x < 1$
Range of f: $-1 < y < 1$
Range of f^{-1}: all real numbers

37.

x	0	1	2	4
$f(x)$	1	2	3	4

x	1	2	3	4
$f^{-1}(x)$	0	1	2	4



39. (a) Proof

(b) $y = \frac{20}{7}(80 - x)$
x: total cost
y: number of pounds of the less expensive commodity

(c) $[62.5, 80]$ (d) 20 lb

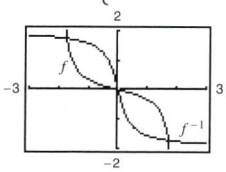
41. Inverse exists. **43.** Inverse does not exist.

45. Inverse exists. **47.** $f'(x) = 2(x - 4) > 0$ on $(4, \infty)$

49. $f'(x) = -8/x^3 < 0$ on $(0, \infty)$

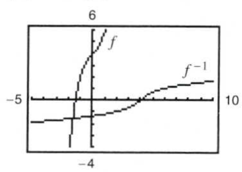
51. $f'(x) = -\sin x < 0$ on $(0, \pi)$

53. $f^{-1}(x) = \begin{cases} \left[1 - \sqrt{1 + 16x^2}\right]/(2x), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$



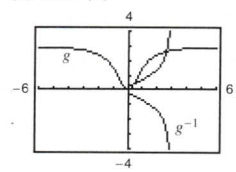
The graph of f^{-1} is a reflection of the graph of f in the line $y = -x$.

55. (a) and (b)



(c) f is one-to-one and has an inverse function.

57. (a) and (b)



(c) g is not one-to-one and does not have an inverse function.

59. One-to-one
$f^{-1}(x) = x^2 + 2,\ x \geq 0$

61. One-to-one
$f^{-1}(x) = 2 - x,\ x \geq 0$

63. $f^{-1}(x) = \sqrt{x} + 3,\ x \geq 0$ (Answer is not unique.)

65. $f^{-1}(x) = x - 3,\ x \geq 0$ (Answer is not unique.)

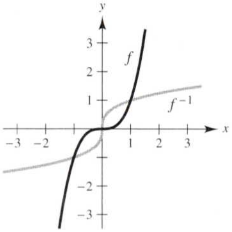
67. Inverse exists. Volume is an increasing function, and therefore is one-to-one. The inverse function gives the time t corresponding to the volume V.

69. Inverse does not exist. **71.** $1/27$ **73.** $1/5$

75. $2\sqrt{3}/3$ **77.** -2 **79.** $1/13$

81. (a) Domain of f: $(-\infty, \infty)$ (b) Range of f: $(-\infty, \infty)$
Domain of f^{-1}: $(-\infty, \infty)$ Range of f^{-1}: $(-\infty, \infty)$

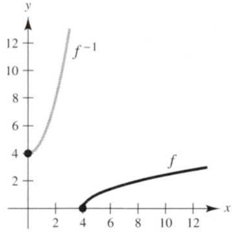
(c)



(d) $f'\left(\frac{1}{2}\right) = \frac{3}{4}$, $(f^{-1})'\left(\frac{1}{8}\right) = \frac{4}{3}$

83. (a) Domain of f: $[4, \infty)$ (b) Range of f: $[0, \infty)$
Domain of f^{-1}: $[0, \infty)$ Range of f^{-1}: $[4, \infty)$

(c)



(d) $f'(5) = \frac{1}{2}$, $(f^{-1})'(1) = 2$

85. $-\frac{1}{11}$ **87.** 32 **89.** 600

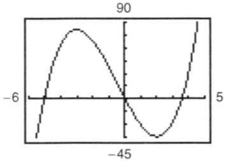
91. $(g^{-1} \circ f^{-1})(x) = (x + 1)/2$ **93.** $(f \circ g)^{-1}(x) = (x + 1)/2$

95. Let $y = f(x)$ be one-to-one. Solve for x as a function of y. Interchange x and y to get $y = f^{-1}(x)$. Let the domain of f^{-1} be the range of f. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
Example: $f(x) = x^3$; $y = x^3$; $x = \sqrt[3]{y}$; $y = \sqrt[3]{x}$; $f^{-1}(x) = \sqrt[3]{x}$

97. Many x-values yield the same y-value. For example, $f(\pi) = 0 = f(0)$. The graph is not continuous at $[(2n - 1)\pi]/2$ where n is an integer.

99. $\frac{1}{4}$ **101.** False. Let $f(x) = x^2$. **103.** True

105. (a)



(b) $c = 2$

f does not pass the horizontal line test.

107–109. Proofs **111.** Proof; concave upward

113. Proof; $\sqrt{5}/5$

115. (a) Proof (b) $f^{-1}(x) = \dfrac{b - dx}{cx - a}$

(c) $a = -d$, or $b = c = 0$, $a = d$